Year:

Attempt all questions.

Group A **(10 x 2 = 20)**

1. Define one-to-one and onto functions with suitable examples.
2. Show by integral test that the series $\sum_{n=1}^{\infty} \frac{1}{x^p}$, converges if p>1.
3. Test the convergence of the series
$$\sum_{n=1}^{\infty} (-1)^{x+1} \frac{1}{x^2}$$
4. Find the focus and the directrix of the parabola $y^2 = 10x$.
5. Find the point where the line $X = \frac{8}{3} + 2t$, $y = -2t$, $z = 1 + t$ intersects the plane $3x + 2y + 6z = 6$.
6. Find a spherical coordinate equation for the sphere $X^2 + y^2 + (z-1)^2 = 1$.
7. Find the area of the region R bounded by $y = x$ and $y = x^2$ in the first quadrant by using double integrals.
8. Define Jacobian determinant for $X = g(u, v, w)$, $y = h(u, v, w)$, $z = k(u, v, w)$.
9. Find the extreme values of $f(x, y) = x^2 + y^2$.
10. Define partial differential equations of the second order with suitable examples.

Group B **(5 x 4 = 20)**

11. State Rolle's Theorem for a differential function. Support with examples that the hypothesis of theorem are essential to hold the theorem.
12. Test if the following series converges
    (a) $\sum_{n=1}^{\infty} \frac{x^2}{2^x}$
    (b) $\sum_{n=1}^{\infty} \frac{2^x}{x^2}$
13. Obtain the polar equations for circles through the origin centered on the x – and y – axis and radius a.
14. Show that the function $f(x) = \begin{cases} \frac{2xy}{x^2+y^2}, (x,y) \neq (0,0) \\ 0, \quad (x,y) = 0 \end{cases}$

    is continuous at every point except the origin.
15. Find the solution of the equation $\frac{\partial 2y}{\partial x2} - \frac{\partial 2z}{\partial y2}$ =x –y.

<u>Group C</u> **(5 x 8 = 40)**

16. Find the area of the region enclosed by the parabola y = 2 –x^2 and the line y =-x.

    OR

    Evaluate the integrals
    (a) $\int_0^3 \frac{dx}{(x-1)^{2/3}}$
    (b) $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$
17. Define a curvature of a space curve. Find the curvature for the helix
    r (t) = (a cost)I + (a sint) j + btk(a, b $\geq$0, a^2 + $b^2 \neq$ 0).
18. Find the volume of the region D enclosed by the surfaces z = x^2 + $3y^2$ and z = 8-x^2 – y^2.
19. Find the maximum and minimum values of the function $f(x,y)$ = 3x + 4y on the circle x^2 + y^2 =1.

    OR

    State the conditions of second derivative test for local extreme values. Find the local extreme values of the function $f(x,y)$ = x^2+ xy + y^2 + 3x – 3y + 4.
20. Define one- dimensional wave equation and one-dimensional heat equations with initial conditions. Derive solution of any one of them.